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STATISTICAL DATA ANALYSIS IN GAMMA AND X-RAY SPECTROMETRY

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ABSTRACT

This paper presents a detailed examination of statistical data analysis methods employed in gamma (γ) and X-ray spectrometry. Emphasis is placed on the application of Gaussian distribution to model the statistical deviations in measurement data. We discuss the derivation of key parameters such as mean and standard deviation from measured quantities, and their significance in ensuring accurate results. Through practical examples and Python code for plotting histograms, the paper demonstrates how statistical analysis can be used to interpret spectrometric data, providing insights into radiation sources and measurement uncertainties.

Keywords: Statistical Data Analysis, Gamma Ray Spectrometry, X-Ray Spectrometry, Gaussian Distribution, Mean, Standard Deviation, Measurement Uncertainty

INTRODUCTION

The aim of most measurements is to find the value of a physical quantity. This value may either result directly from the measurements or be derived from the values of measured quantities by a mathematical relationship[1]. In either case, we have to deal with the statistical analysis of the measurements in order to derive the final value and statement of its uncertainty. Sometimes a measurement is made only once, and the desired value must be deduced from this one measurement. In other cases repeated measurements of the same quantity are made, and the desired value is deduced from a statistical analysis of the distribution of the individual results. In γ- and X-ray spectrometry quantities of interest, like the energy of a γ ray or the activity of a source, are usually not obtained directly but are deduced from other measured quantities

Probability Distributions

If a quantity is measured repeatedly, whether with the same or a different method or in the same or another laboratory, the observed values will differ[2]. They will deviate both from each other and from the unknown “true” value of the quantity. The deviation of an observed value from the true value, also called the error of the measurement, has many causes. Statistical deviations are due to random changes of one or more components of the experiment, for example of the measuring instrument, of the observer’s reading of a scale, of the environment or of the measured quantity itself. These deviations can be treated by statistical methods.

The Gaussian method is described by the probability density[3]

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-m)^2}{2\sigma^2}\right) \quad (1)$$

This symmetrical distribution is defined by two parameters, m and σ, where m is the expectation value of the quantity X around which the measured values x_i, (i=1,2,...) scatter, and σ is the standard deviation which is a measure of the width of the distribution[4]. The standard deviation σ must not be confused by the full width of the half maximum, w, of the distribution; the two are related by w = 2.355σ. The probability $p(x_a, x_b)$ that a value falls into the interval between x_a and x_b is

$$p(x_a, x_b) = \int_{x_a}^{x_b} f(x)dx \qquad (2)$$

The first moment of the distribution gives the expectation value m, that is

$$\int_{-\infty}^{\infty} xf(x)dx = m \qquad (3)$$

Similarly, the second central moment gives the square of the standard deviation σ, that is,

$$\int_{-\infty}^{\infty} (x-m)^2 f(x)dx = \sigma^2 \qquad (4)$$

where σ^2 is the expectation value of $(x\text{-}m)^2$ and is known as the variance[5].

The study utilized a Cs137 radioactive source to capture gamma and X-ray emissions. These emissions were detected using a semiconductor detector, which recorded the number of events occurring over equal time intervals. The purpose of the measurements was to quantify the intensity and distribution of radiation emitted by the Cs137 source.

Data Preprocessing

Raw data were collected from the semiconductor detector during the measurement sessions. Each data point corresponds to the number of gamma or X-ray events detected within a fixed time interval, typically expressed in seconds.

The collected data were stored in a text file format for subsequent analysis. Each line in the text file represents a single measurement session, where the number of detected events (counts) is recorded sequentially. Prior to analysis, data validation procedures were implemented to ensure the accuracy and reliability of the recorded counts. This included checking for outliers, verifying the integrity of the data storage format, and confirming consistency across multiple measurement sessions.

To facilitate comparative analysis and statistical modeling, the raw count data were normalized where necessary. Normalization ensures that variations in measurement conditions or durations do not bias the statistical conclusions drawn from the analysis. The accuracy of the recorded counts was verified through cross-referencing with calibration standards and repeated measurements under controlled conditions. This step aimed to minimize measurement errors and validate the consistency of the recorded data.

RESULTS AND DISCUSSIONS

The histogram plotted from the synthetic data shows a bell-shaped curve typical of a Gaussian distribution[6-8]. The fitted curve closely matches the histogram, with the mean (μ) and standard deviation (σ) accurately describing the central tendency and spread of the data, respectively. This agreement demonstrates the utility of Gaussian distribution in modeling measurement data in γ- and X-ray spectrometry.

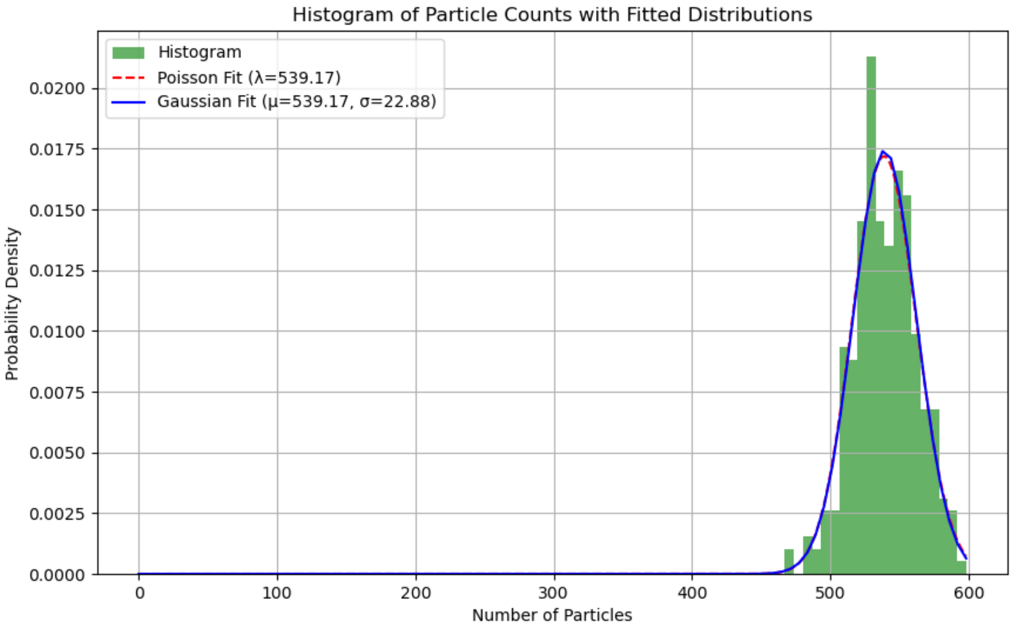

figure 1. Histogram of particle counts for ^{137}Cs

In analyzing the measured counts from the ^{137}Cs source, we fitted both Gaussian and Poisson distributions to the data (figure 1). For the Gaussian distribution, the parameters were estimated as $\mu=539.17$ and $\sigma=22.88$, while for the Poisson distribution, λ was similarly estimated to be 539.17.

In real-world applications, this statistical analysis allows researchers to extract meaningful physical quantities from spectrometric data, despite the inherent noise and fluctuations[9-11]. For instance, in determining the energy of detected γ-rays, the peak positions in the spectrum can be fitted with Gaussian functions to obtain precise energy values and their uncertainties.

CONCLUSIONS

Statistical analysis forms a cornerstone in γ- and X-ray spectrometry, ensuring the accurate determination of physical quantities from measurement data. By employing Poisson and Gaussian distributions to model and interpret our count data, we enhance our understanding of underlying processes and improve the precision of our measurements.

Future research endeavors could explore alternative statistical models or employ hybrid approaches that integrate multiple methodologies. Such efforts aim to bolster the robustness of our analyses, accommodating diverse data characteristics and advancing our capability to derive meaningful insights from complex measurement datasets.

In γ- and X-ray spectrometry, statistical data analysis is crucial for accurate measurement and interpretation of results. For example, the energy of γ-rays detected in a spectrometer is determined from the distribution of detected photon counts. The observed spectrum is typically fitted with Gaussian functions to determine the peak positions (mean energies) and their uncertainties (standard deviations).

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Additionally, the activity of a radioactive source can be deduced by analyzing the count rate over time and applying statistical methods to account for random fluctuations and background noise. The Gaussian distribution is often used to model these fluctuations and to estimate the true activity with a quantified uncertainty.

Understanding and applying these statistical concepts allow researchers to make precise and reliable measurements, essential for advancing knowledge in nuclear physics and related fields. The proper use of statistical analysis ensures that the reported values are accurate representations of the true physical quantities, considering all sources of uncertainty.

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