GALAXY INTERNATIONAL INTERDISCIPLINARY RESEARCH JOURNAL (GIIRJ)
ISSN (E): 2347-6915
Vol. 11, Issue 11, November (2023)

INVERSE TRIGONAMETRIC FUNCTIONS AND RELATIONSHIPS BETWEEN THEM

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ABSTRACT

Methods for solving equation invdving inverse trigonometric unctions are studied. One of the most popular ways to solve equations involving inverse trigonometric functions is to perform a trigonometric operation on both sides of a given equation. This solution method is explained by creating an equation equivalent to this equation.

ANNOTATSIYA

Teskari trigonametrik funksiyalar ishtirok etgan tenglamalar yechish usullari o`rganilgan. Teskari trigonometrik funksiyalar qatnashgan tenglamalarni yechishning eng ommalashgan usullaridan biri berilgan tenglamaning har ikki tomonida biror trigonometrik operasiyani bajarish orqali bajariladi. Yechishning bu usuli berilgan tenglamaga ekvivalent tenglama hosil qilish bilan izlanadi.

АННОТАЦИЯ

Изучены методы решения уравнений, описывающих обратные тригонометрические функции. Один из самых популярных способов решения уравнений, включающих обратные тригонометрические функции, - это выполнение тригонометрической операции с обеими сторонами данного уравнения. Этот метод решения объясняется созданием уравнения, эквивалентного данному уравнению.

Consider the following simple equations:

$$arcsinx = m \qquad arccosx = m$$

$$arctgx = m \qquad arcctgx = m$$

This is one of the equations $arcsinx = m$ Let's take a closer look at the equation. Area of values of arcsin $\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$ dan iborat bo‘lgani uchun bu tenglama $|m| \le \frac{\pi}{2}$ will have a solution when This is the only solution under this condition $x = sinm$ will be.

The remaining equations are also seen.

If $0 \le m \le \pi$ if $arccosx=m$ the equation is unique $x=cosm$ will have a solution.

If m $[0;\pi]$ if it does not belong to the interval, there is no solution.

If $m \epsilon(-\frac{\pi}{2};\frac{\pi}{2})$ if it belongs to the interval, $arctgx = m$ the equation is unique $x=tgm$ has a solution.

If $m \epsilon(0;\pi)$ if it belongs to the interval, $arcctgx = m$ the equation is unique $x= ctgm$ has a solution.

f(arcsinx)=0 solution of the equation (*f* the following mixed system can be solved by substituting under the function:

$$\begin{cases} f(t)=0 \\ -\frac{\pi}{2} \le t \le \frac{\pi}{2} \end{cases}$$

f(arcsinx, arccosx)=0 solving the equation $arccosx=\frac{\pi}{2}-arcsinx$ is used to solve the above equation.

$f(x) = g(x)$ (1)

Let's look at the equation. $sinf(x)=sing(x)$ (2)

(2) equation (1) is the result of the equation, the converse is not true. Equation (2) in turn

$f(x)=(-1)^n g(x)+\pi n$ (3)

equivalent to Eq.

Any of (3). $n\neq 0$ is an odd root for equation (1) when

Equation (1) is also $tgf(x)=tgg(x)$ (3) is equivalent to Eq.

$$f(x)=g(x)+\pi n \quad (4)$$

equation $n\neq 0$ any solution in is a solution to equation (3), but not to equation (1).

Thus, when we pass from equation (1) to equation (4), roots may disappear or extraneous roots may appear.

- For example, $x=\pi-x$ (1) the equation is unique $x=\frac{\pi}{2}$ has a solution.

$tg(x)=tg(\pi-x)$ (2) equation $x=\pi k$ has a solution. Extraneous roots in the transition from equation (1) to (2). $x = \pi k$ is formed $x = \frac{\pi}{2}$ the solution disappears.

In many cases, an algebraic equation is formed when we perform a trigonometric operation on both sides of an equation involving arc functions. In any such cases, all the roots of the given equation are among the roots of the algebraic equation, and there may be cases where some roots may disappear when we pass from equation (1) to equation (4). As a result, the solution of the given equation it is enough to find all the roots of the algebraic equation in the field of real numbers and check by substituting them into the given equation.

An algebraic equation formed after a trigonometric operation is performed on both sides of an equation involving given arc functions is irrational. In order to form an algebraic equation, it is necessary to save the irrational equation from radicals, which in turn may produce extraneous roots. Another source of the appearance of extraneous roots is real form substitutions.

For example, *arcsinf(x)=arcsing(x)* the set of possible values of the unknown in the equation is determined by 2 conditions, *x* value of *f(x)* and *g(x)* should belong to the field of detection.

1. The following $|f(x)| \leq 1$, $|g(x)| \leq 1$ inequalities must be fulfilled.

f(x)=g(x) when we pass to the equation (if this last equation is considered unconnected with the given equation), the 2nd condition is dropped.

The change in the set of possible values of the unknown is justified by the following specific substitutions:

$$sin(arcsinf(x)) = f(x)$$

$$sin(arcsing(x)) = g(x)$$

Examples.

Example 1. $3arcsin\sqrt{x} - \pi = 0$ **solve the equation.**

Solution: $arcsin\sqrt{x} = \frac{\pi}{3}$; $\sqrt{x} = \frac{\sqrt{3}}{2}$; $x = \frac{3}{4}$.

Example 2. $4arctg(x^2\text{-}3x+3)\text{-}\pi=0$ **solve the equation.**

Solution: $$arctg(x^2-3x+3) = \frac{\pi}{4}$$

$$x^2-3x+3 = 1$$

$$x_1=1 \quad x_2=2$$

Example 3. $2arcsinx=8$ **solve the equation.**

Solution: The equation has no solution because $arcsinx = u$ bo‘lib $u > \frac{\pi}{2}$.

Example 4. $\pi\text{-}arcsinx = arccosx$ **(1) solve the equation.**

Solution: $$sin(\pi-arcsinx) = sin(arccosx)$$

$$sin(arcsinx) = sin(arccosx)$$

$$x = \sqrt{1-x^2} \quad (2)$$

$$2x^2 = 1; \quad x = \pm\frac{1}{\sqrt{2}}$$

$-\frac{1}{\sqrt{2}}$ (2) is not a root of an irrational equation. This is the square root of (2) when we square both sides.

$x=\frac{1}{\sqrt{2}}$ the irrational equation is a solution of (2) and is not a solution of the given equation (1). Hence, equation (1) has no solution, otherwise

$$arcsinx+arccosx = \frac{\pi}{2}$$

a contradiction arises in reality.

$x = \frac{1}{\sqrt{2}}$ another $arcsinx = arccosx$ satisfies Eq.

Example 5. $arctg(x+2) - arctg(x+1) = \frac{\pi}{4}$ **solve the equation.**

Solution: Tangent both sides of the equation $x^2+3x+2=0$

we form the equation $x_1= -2$, $x_2 = -1$

Both roots satisfy the given equation.

Example 6. $arccosx = arctgx$ solve the equation.

Solution: Cosine both sides of Eq

$$x = \frac{1}{\sqrt{x^2+1}}$$ we generate .

From this $x^2(x^2+1) = 1$ yoki $x^4+x^2-1 = 0$

$$x_1=\sqrt{\frac{\sqrt{5}-1}{2}} \quad va \quad x_2 =-\sqrt{\frac{\sqrt{5}-1}{2}}$$

The first of these roots satisfies the given equation.

Example 7. $2arcsinx = arccos2x$ **(1) solve the equation.**

Solution: $cos(2arcsinx) = cos(arccos2x)$

$$1-2x^2 =2x$$

$$2x^2+2x-1=0 \quad (2)$$ As a result

$$x_1 = \frac{-1-\sqrt{3}}{2}, \quad x_2 = \frac{-1+\sqrt{3}}{2}$$

(1) is the definition domain of Eq $\begin{cases} |x| \le 1 \\ |2x| \le 1 \end{cases}$ from that $-\frac{1}{2} \le x \le \frac{1}{2}$.

This condition x_2 qit will make you happy . x_1 extraneous root resulted from the expansion of the field of detection when we pass from equation (1) to equation (2).

Example 8. ***arsin mx = arcos nx*** **solve the equation.**

Solution: *sin(arsin mx)=sin(arccos nx)*

From this we create the following irrational equation.$mx = \sqrt{1-n^2x^2}$

$$m^2x^2 = 1 - n^2x^2$$

$$(x^2+m^2)\,x^2 = 1$$

ago $x = \pm\frac{1}{\sqrt{m^2+n^2}}$ the following cases may occur.

1. $m \geq 0, n \geq 0$ bo'lib, Let m or n be non-zero. In this case, the equation is satisfied only for positive values of x.

2. $x \leq 0$ da esa *arcsinmx* and *arccosnx* the arcs are spaced at different intervals. The only solution of Eq $x = \frac{1}{\sqrt{m^2+n^2}}$

3. $m \leq 0, n \leq 0$ to be, m yoki *n 0* be different from In this case, the equation will not have positive solutions. Its only solution $x = -\frac{1}{\sqrt{m^2+n^2}}$

4. $m > 0, n < 0$. U The equation does not have a solution without Arcfunctions *mx* and *nx* arguments are of different sign, so *arcsinmx* and *arccosnx* the arcs are spaced at different intervals. $m < 0$, $n > 0$ the same is the case.

5. $m = n = 0$ bu holda tenglama ziddiyatli.

Example 9. ***arcsin2x+arcsinx*** $= \frac{\pi}{3}$ **(1) solve the equation.**

Solution: $cos(arcsin2x+arcsinx)=\cos\frac{\pi}{3}$

we form an irrational equation.

$$\sqrt{1-4x^2}\sqrt{1-x^2} - 2x^2 = \frac{1}{2} \quad (2)$$

$$28x^2 - 3 = 0$$

$x = \pm\frac{1}{2}\sqrt{\frac{3}{7}}$ $x = -\frac{1}{2}\sqrt{\frac{3}{7}}$ value will not be a root of the given equation.

$arcsin(-\sqrt{\frac{3}{7}})$ and $arcsin(-\frac{1}{2}\sqrt{\frac{3}{7}})$ arcs $(-\frac{\pi}{2};0)$ belongs to the interval and is their sum $\frac{\pi}{3}$ will not be equal to $x = \frac{1}{2}\sqrt{\frac{3}{7}}$ value will be the root of the given equation.

Example 10. $\begin{cases} arcsinx\ arcsiny = \frac{\pi^2}{12} \\ arccosx\ arccosy = \frac{\pi^2}{24} \end{cases}$ **remove the system.**

Solution: $arccosa = \frac{\pi}{2} - arcsina$ we express the left part of the equation 2 by the arcsine using the equation.

$$(\frac{\pi}{2} - arcsinx)(\frac{\pi}{2} - arcsiny) = \frac{\pi^2}{24}$$

u = arcsinx, v = performing arcsiny substitutions,

$\begin{cases} uv = \frac{\pi^2}{12} \\ uv - (u+v)\frac{\pi}{2} + \frac{5\pi^2}{24} = 0 \end{cases}$ we create a system.

$u+v$ va uv , whose roots are equal to u and v

$$12z^2 - 7\pi z + \pi^2 = 0$$

we form the equation From this: $u_1 = \frac{\pi}{3}$, $v_1 = \frac{\pi}{4}$ and $u_2 = \frac{\pi}{4}$, $v_2 = \frac{\pi}{3}$

As a result, we create a combination of 2 systems:

$$\begin{cases} arcsinx = \frac{\pi}{3} \\ arcsiny = \frac{\pi}{4} \end{cases} \text{ va } \begin{cases} arcsinx = \frac{\pi}{4} \\ arcsiny = \frac{\pi}{3} \end{cases}$$

The given system has 2 solutions:

$$x_1 = \frac{\sqrt{3}}{2},\ y_1 = \frac{\sqrt{2}}{2}.$$

$$x_2 = \frac{\sqrt{2}}{2},\ y_2 = \frac{\sqrt{3}}{2}.$$

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